

USA Mathematical Talent Search

PROBLEMS

Round 1 - Year 13 - Academic Year 2001-2002

1/1/13. Determine the unique positive two-digit integers m and n for which the approximation

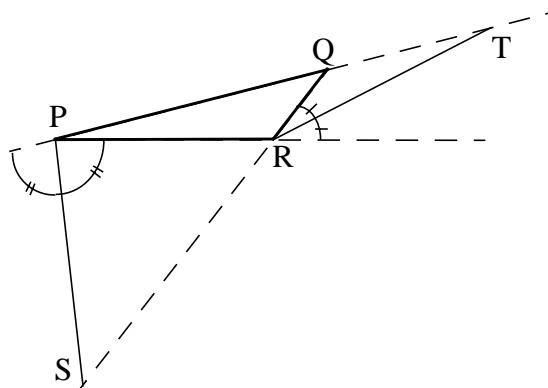
$$\frac{m}{n} \approx 0.2328767 \text{ is accurate to seven decimals, i.e., } 0.2328767 \leq \frac{m}{n} < 0.2328768.$$

2/1/13. It is well known that there are infinitely many triples of integers (a, b, c) whose greatest common divisor is 1 and which satisfy the equation $a^2 + b^2 = c^2$. Prove that there are also infinitely many triples of integers (r, s, t) whose greatest common divisor is 1 and which satisfy the equation $(rs)^2 + (st)^2 = (rt)^2$.

3/1/13. Suppose $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle x , $0 \leq x \leq \frac{\pi}{2}$. Determine $\frac{\sin 3x}{\sin x}$ for the same x .

4/1/13. The *projective plane* of order three consists of 13 “points” and 13 “lines”. These lines are not Euclidean straight lines; instead they are sets of four points with the properties that each pair of lines has exactly one point in common, and each pair of points has exactly one line that contains both points. Suppose the points are labeled 1 through 13, and six of the lines are $A = \{1, 2, 4, 8\}$, $B = \{1, 3, 5, 9\}$, $C = \{2, 3, 6, 10\}$, $D = \{4, 5, 10, 11\}$, $E = \{4, 6, 9, 12\}$, and $F = \{5, 6, 8, 13\}$. What is the line that contains 7 and 8?

5/1/13. In $\triangle PQR$, $QR < PR < PQ$ so that the exterior angle bisector through P intersects ray \overrightarrow{QR} at point S , and the exterior angle bisector at R intersects ray \overrightarrow{PQ} at point T , as shown on the right. Given that $PR = PS = RT$, determine, with proof, the measure of $\angle PRQ$.



Complete, well-written solutions to at least two of the problems above, accompanied by a **Cover Sheet** and **Entry Form** should be sent to the address listed on the USAMTS web site

<http://www.nsa.gov/programs/mepp/usamts.html>

and **postmarked no later than 7 October 2001**. Each participant is expected to develop solutions without help from others.

